

XI. *On the Parallax of the Fixed Stars.* By Mr. Herchel, F. R. S.; communicated by Sir Joseph Banks, Bart. P. R. S.

Read December 6, 1781.

TO find the distance of the fixed stars has been a problem which many eminent astronomers have attempted to solve; but about which, after all, we remain in a great measure still in the dark. Various methods have been pursued without success, and the result of the finest observations has hardly given us more than a distant approximation, from which we may conclude, that the nearest of the fixed stars cannot be less than forty thousand diameters of the whole annual orbit of the earth distant from us. Trigonometry, by whose powerful assistance the mathematician has boldly ascended into the planetary regions, and measured the diameters and orbits of the heavenly bodies, for want of a proper base, can here be but of little service; for the whole diameter of the annual orbit of the earth is a mere point when compared to the immense distance of the stars. Now, as it is not in our power to enlarge this base, we can only endeavour to improve the instruments by which we measure its parallax.

There are two things requisite for measuring extremely small angles with accuracy. First, that the instrument we use for this purpose, be it quadrant, sector, or micrometer, should be divided and executed with sufficient exactness; and, secondly, that the telescope, by which the observations are to be made,

made, should have an adequate power and distinctness. Upon the first head, the great improvements of mathematical instrument-makers have hardly left us any thing to desire: we can now measure seconds with almost as much facility and truth as former observers could measure minutes; nor do I think it impossible to go still further, and divide instruments that would shew thirds with sufficient accuracy. It is in the latter, or optical part, we find the greatest difficulty. To see a single second of a degree with precision requires a telescope of very great perfection; therefore, supposing the mechanical part of an apparatus well executed, it will still be necessary to try how far the power of our telescope will enable us to ascertain with confidence the division or number of seconds it points out. If upon trial we find that our instrument will give us the same measure within the second, every time the experiment is repeated, we may pronounce it capable of measuring seconds; if otherwise, it will remain to be examined, whether the fault lies in the mechanical or optical part.

Let us now suppose that the parallax of the fixed stars does not amount to a single second, yet still the case is by no means desperate; and though the difficulty of measuring seconds will soon suggest to us what extraordinary powers and distinctness of the telescope, and accuracy of the micrometer, are required to measure thirds; this ought by no means to discourage us in the attempt. Could we measure angles, much smaller than seconds, might we not hope to find the parallax of some of the fixed stars at least to amount to several thirds? On the other hand, if it should appear, indeed, that even with such improved methods of measurement we could not reach the remote situation of such almost infinitely distant suns, we might still derive a valuable approximation towards truth from

such repeated observations, even though they should not be attended with all the success we expected from them. On this assurance, I endeavoured to take such a method for attempting the investigation of the parallax of the stars as to avail myself of the improvements I had already made, and was still in hopes of making, in my telescopes.

The next thing that was necessary to consider in this undertaking was, the manner of putting it into execution. The method pointed out by GALILEO, and first attempted by HOOK, FLAMSTEAD, MOLINEUX, and BRADLEY, of taking distances of stars from the zenith that pass very near it, though it failed with regard to parallax, has been productive of the most noble discoveries of another nature. At the same time it has given us a much juster idea of the immense distance of the stars, and furnished us with an approximation to the knowledge of their parallax that is much nearer the truth than we ever had before. Dr. BRADLEY, in a letter to Dr. HALLEY on the subject of a new discovered motion of the fixed stars, says, “ I believe I may venture to say, that in either of the “ two stars last mentioned (γ Draconis and η Ursæ majoris) it “ (the annual parallax) does not amount to 2”. I am of opinion, that if it were 1” I should have perceived it in the “ great number of observations that I made, especially upon “ γ Draconis; which agreeing with the hypothesis (without “ allowing any thing for parallax) nearly as well when the “ sun was in conjunction with, as in opposition to, this star, “ it seems very probable, that the parallax of it is not so great “ as one single second.” *Phil. Trans.* n. 406. p. 637. Dec. 1728. As I do not know that any thing more decisive has been done upon the subject, it will not be amiss to see how far this method of finding the parallax has really been successful.

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The instrument that was used upon this occasion was the same as the present zenith sectors, which can hardly be allowed sufficient to shew an angle of one or even two seconds with accuracy; yet, on account of the great number of observations, and above all the great sagacity of the observer, we will admit that if the parallax had amounted to two seconds he would have perceived it. The star on which these observations were made is marked of the third magnitude in the catalogue of PTOLEMY; in TYCHO BRAHE's of the third; in the Prince of HESSE's of the third; in HEVELIUS's between the third and second; in FLAMSTEAD's of the second; and now appears as a very bright star of the third, or small star of the second magnitude; therefore its parallax is probably considerably less than that of a star of the first magnitude. Several authors who have touched upon this subject seem to have overlooked this distinction; and from Dr. BRADLEY's account of the parallax of γ Draconis, have concluded the parallax of the stars in general not to exceed $1''$; but this appears to me by no means to follow from the doctor's observations. It is rather evident that, for aught we know to the contrary, the stars of the first magnitude may still have a parallax of several seconds; and I believe this to be as accurate a result as that method is capable of giving, at least in latitudes where there is not a star of the first magnitude that passes directly through the zenith*.

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* DE LA LANDE, in his excellent book of Astronomy, says, that the parallax of the fixed stars has been proved to be absolutely insensible (Astr. liv. XVI. § 2782.). He reports the observations of TYCHO BRAHE, PICARD, HOOK, and FLAMSTEAD, and concludes (§ 2778.) from the discovery of the aberration by Dr. BRADLEY (which it seems he also allows to be the most decisive upon the subject) that now the question about parallax is resolved. In giving us the opinion which

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In general, the method of zenith distances labours under the following considerable difficulties. In the first place, all these distances, though they should not exceed a few degrees, are liable to refractions; and I hope to be pardoned when I say that the real quantities of these refractions, and their differences,

the doctor had of the result of his own observations with regard to the annual parallax, DE LA LANDE only mentions "M. BRADLEY pense que si elle (la "parallaxe) eût été seulement de 1" il l'auroit apperçue dans le grand "nombre d'observations qu'il avoit faites, surtout de γ du Dragon." But if we also take in those lines upon which Dr. BRADLEY seems to lay the greatest stress, viz. "I believe I may venture to say, that in either of the "two stars last mentioned it does not amount to two seconds;" and if we allow for the magnitude of the stars upon which the observations were made, I think I have fairly stated the full amount of all the actual proofs we have of the smallness of the annual parallax. Now, since it has escaped the finest observations of BRADLEY, it is not likely that it should come up to the full quantity to which it might amount without being perceived; and therefore the doctor might think it highly probable, "that it is not so great as one single second;" and his opinion, as well as DE LA LANDE's, who believes it to be absolutely insensible, are perfectly consistent with all the observations that have hitherto been made; though the *actual proofs*, which are the subject of our present inquiry, do not extend so far. Against the parallax of Sirius DE LA LANDE (§ 2781.) mentions "forty- "five meridian altitudes taken by Dr. BEVIS [a], with the eight-feet mural qua- "drant of the Royal Observatory at Greenwich, none of which differed 3 or 4" "from the mean altitude." Now, if they differed 3 or 4" from the mean, we may suppose they differed 6 or 8" from each other; and that observations, subject to so many causes of error as I shall presently enumerate, and which differed so much from each other, cannot give the least evidence either for or against a parallax, will need no proof. Refraction alone, which is liable to such changes at the meridian altitude of Sirius, notwithstanding the most careful observations of the barometer and thermometer should be made to ascertain its quantity, would, with me, remain an unanswerable argument against the validity of such observations in a subject of this critical nicety.

[a] These observations were not made by Dr. BEVIS, but extracted from the registers of the Royal Observatory at my desire, and calculated by myself, and sent in a letter by Dr. BEVIS to Paris.

are very far from being perfectly known. Secondly, the change of position of the earth's axis arising from nutation, precession of the equinoxes, and other causes, is so far from being completely settled, that it would not be very easy to say what it exactly is at any given time. In the third place, the aberration of light, though best known of all, may also be liable to some small errors, since the observations from which it was deduced laboured under all the foregoing difficulties. I do not mean to say, that our theories of all these causes of error are defective; on the contrary, I grant that we are for most astronomical purposes sufficiently furnished with excellent tables to correct our observations from the above mentioned errors. But when we are upon so delicate a point as the parallax of the stars; when we are investigating angles that may, perhaps, not amount to a single second, we must endeavour to keep clear of every possibility of being involved in uncertainties; even the hundredth part of a second becomes a quantity to be taken into consideration.

I shall now deliver the method I have taken, and shew that it is free from every error to which the former is liable, and is still capable of every improvement the telescope and mechanism of micrometers can furnish.

Let OE (fig. 1.) be two opposite points of the annual orbit, taken in the same plane with two stars a, b , of unequal magnitudes. Let the angle aOb be observed when the earth is at O : and let the angle aEb be also observed when the earth is at E . From the difference of these angles, if any should be found, we may calculate the parallax of the stars, according to a theory that will be delivered hereafter. These two stars, for reasons that will soon appear, ought to be as near each other as possible,

possible, and also to differ as much in magnitude as we can find them.

GALILEO, I believe, was the first who suggested this method; but in the manner he mentions it in his third dialogue of the *Systema Cosmicum*, it would be exposed to all the difficulties we have enumerated, and would wish to avoid; for he does not observe, that the two stars should be so near each other as thereby to preclude the influence of every cause of error.

This method has also been mentioned by other authors; and we find that Dr. LONG observed the double star which is the first of Aries in PTOLEMY's catalogue; that in the head of Castor; the middle one in the sword of Orion; and that in the breast of Virgo, with telescopes of fourteen and seventeen feet, and "was persuaded they would be found always to appear the same." But when the theory of parallax will be explained, it will be seen that every one of these stars are totally improper for the purpose; for the stars of γ Arietis are near 10'' distant from each other, and moreover equal in magnitude. In α Geminorum the stars, though near enough, do not sufficiently differ in magnitude to shew any parallax. The stars in the Nebula of Orion, on account of their extreme smallness or distance, are still more improper than any; and those of γ Virginis are equal in magnitude.

I do not find that any thing else has been done upon the subject. GALILEO justly remarks, that such observations ought to be made with the best telescopes, and upon this occasion mentions the power of his own, which enlarged the disk of the sun a thousand times, from which we find it magnified about thirty-two times; but we can hardly think his nor even Dr. LONG's, whose power might probably be sixty or seventy, sufficient for the purpose. What would GALILEO say, if he were told that

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our present opticians make instruments that enlarge the disk of the sun above forty thousand times? What would even CASSINI say, if he were to view the first star of Aries, which appeared to him as split in two, through a telescope that will shew η Coronæ borealis and b Draconis to be double stars?

But to proceed, I shall now prove that this method, if stars properly situated (such as I have found) are taken, is free from all the errors occasioned by refraction, nutation, precession of the equinoxes, changes of the obliquity of the ecliptic, and aberration of light; and that the annual parallax, if it even should not exceed the tenth part of a second, may still become visible, and be ascertained at least to a much greater degree of approximation than it ever has been done.

It will also appear, from the great number of observations I have already made upon several double stars, especially ϵ Bootis, that we can now with much greater certainty affirm the annual parallax to be exceedingly small indeed; and that there is a great probability of succeeding still farther in this laborious but delightful research, so as to be able at last to say, not only how much the annual parallax *is not*, but how much it really *is*.

Let there be two stars at a distance from each other, not exceeding five seconds; suppose them to be observed at an altitude of 20° ; and let them be so situated with respect to each other, that one of them may be 20° , and the other 20° and $5'$ high: then the whole effect of mean refraction at that altitude, by Dr. MASKELYNE's excellent tables, will be $2' 35''{,}5$ for 20° , and $2' 35''{,}4888$ for $20^\circ 5''$. The difference is $0''{,}0111$. Now, in the first place, we have nothing to do with the refraction itself, since the real altitude of the stars is not in question. In the next place, we also have no concern with the difference of refraction

tion between the two stars, though no more than the ,0111th part of a second, because the real distance between the two stars is not required. It follows then, that these observations can only be affected by the difference of the difference; that is, by an alteration in the quantity of refraction occasioned by the change of heat and cold, or weight of the atmosphere, and pointed out to us by the rise and fall of the barometer and thermometer. Let us then see what this difference of the difference may amount to. Suppose a change of 22° of FAHRENHEIT'S thermometer, that is, from the freezing point to the moderate air of a summer's night, and a difference of an inch in the height of the barometer; these two causes both conspiring, which does not often happen, may occasion an alteration of ,00096th part of a second in five, at an altitude of 20° ; but this being less than the thousandth part of a second may safely be rejected as a quantity altogether insensible.

Since it may not be always convenient to view those stars at the altitude of 20° , it remains to see what effect different altitudes may have: let us then make the most unfavourable supposition, that they may one time be seen in a horizontal position, having before been seen vertical. In this case, as the whole difference of refraction in a difference of $5''$ of altitude is no more than ,0111, provided they are observed not lower than 20° , and the whole difference of the difference of refraction is only ,0009; the sum ,012, when both conspire, not exceeding much the hundredth part of a second, may still be rejected as insensible. Let us also examine how near the horizon it may be safe to observe such stars. At 10° , for instance, the refraction is $5' 14'',6$; the difference for $5''$ is ,0388; the joint effect of the changes in the barometer and thermometer is ,0034; the sum of the whole together amounts to ,0422, which

is less than half the tenth of a second: now this may either be taken into consideration, or such low observations may be avoided, as being by no means necessary, and but ill suiting the high powers a telescope proper for this purpose ought to bear.

The change of position of the earth's axis I look upon as an unfurmountable obstacle to taking the parallax of stars by the method of zenith distances: for though refraction is much reduced in the zenith, this change is there no less sensible than in other parts of the heavens; but as this will always affect our two stars exactly alike, we are entirely freed from this embarrassment.

The aberration of light can have no influence of the least consideration upon our two stars, as a mere inspection of the tables will shew. In a whole degree, its effects, when greatest, amount but to four-tenths of a second, and consequently in $5''$ to no more than ,0005, or the two thousandth part of a second.

Observations of the relative distance of the two stars that make up a double star, being thus cleared of every impediment, are capable of being continually improved by every degree of perfection the telescope may acquire: we can chuse stars that may be viewed sufficiently high to be clear of the vapours that swim near the horizon, and consequently employ the greatest powers our instruments are capable of. From experience I can also affirm, that the stars will bear a much higher degree of magnifying than other celestial objects. Too much has hitherto been taken for granted in optics: every natural philosopher is ready enough to allow the necessity of making experiments, and tracing out the steps of nature; why this method should not be more pursued in the art of seeing

does not appear. Theories are only to be used when proper data are assigned; but the data are carefully to be re-examined, when new improvements may widely alter the result of former experiments. Thus, we are told, that we gain nothing by magnifying *too much*. I grant it; but shall never believe I magnify too much till by experience I find, that I can see better with a lower power. Nor is even that sufficient: a lower power may shew more of the object; it may shew it brighter, nay even distincter, and therefore upon the whole better; and yet the greater power may, in a particular case, be preferable: for if the object is so small as not to be at all visible with the lower power, and I can, by magnifying more, obtain a view of it, though neither so bright nor distinct as I could wish, is it not evident, that here this power is preferable to the former?

The naturalist does not think himself obliged to account for all the phænomena he may observe; the astronomer and optician may claim the same privilege. When we increase the power we lessen the light in the inverse ratio of the square of the power; and telescopes will, in general, discover more small stars the more light they collect; yet with a power of 227 I cannot see the small star near the star following *o* Aquilæ, when, by the same telescope, it appears very plainly with the power of 460: now, in the latter case, the power being more than double, the light is less than the fourth part of the former. In such particular cases I generally suspect my own eyes, and have recourse to those of my friends. I had the pleasure of shewing this star to Dr. WATSON junior, who soon discovered the small star, which accompanies the other, with the power of 460; but saw nothing of it with 227, though the place where to look for it had been pointed out to him by the higher power. The experiment has been too often
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repeated to be doubtful, and has also been confirmed by others of nearly the same nature: for instance, the smallest of the two that accompany the star near k Aquilæ, the small star near μ Herculis, and the small star near α Lyræ, are invisible with my power of 227, and visible with the same aperture when the power is 460. Also the small stars near FLAMSTEAD'S 24th of Aquila, the smallest of two near σ Coronæ, the small star near the star south of ϵ Aquilæ, the small star near the second ν Persei, the small star near the star which accompanies FLAMSTEAD'S 10th sub pede et scapula dextra Tauri, the small star, near β Delphini, and the small star near the pole star, are all much brighter and stronger, and therefore much sooner seen with 460 than with 227.

Great power may also, in particular circumstances, be favourable, even with an excess of aberration. When two stars are so close together as to make the scale for measuring the distance of their centers too small, if, by magnifying much, we can enlarge that distance, we may gain a considerable advantage, provided the centers or apparent bodies of the stars remain distinct enough for the purpose of these measures. The appearance of α Lyræ in my Newtonian reflector with a power of 460 is represented in fig. 2.; with 2010 in fig. 3.; with 3168 in fig. 4.; and with 6450 in fig. 5. Now in all these figures we see, that the centers are still distinct enough to measure their distances with sufficient truth; or if any little error should be introduced by the magnitude of the central point, it will be more than sufficiently balanced by the largeness of the scale. In this manner, with a power of 3168, I have obtained a scale of no less than ten inches six tenths for the distance of the centers of the two stars of α Geminorum; and as we

know these centers to be but a few seconds distant, it is plain how great an advantage we gain by such an enlarged scale.

These experiments have but very lately pointed out to me a method of making a new micrometer, upon a construction entirely different from any that are now in use, which I have been successful enough to put in practice, and by which I have already begun to determine the distance of the centers of some of the most remarkable double stars to a very great degree of accuracy*.

The powers that may be used upon various double stars are different, according to their relative magnitudes: ϵ Bootis, for instance, will not bear the same power as α Geminorum, nor would it be difficult to assign a reason for it; but as I here shall merely confine myself to facts, it will be sufficient in general to mention, that two stars, which are equal, or nearly so, will bear a very high power: with α Geminorum I have gone as far as 3168; but with the former only to 2010. The difficulty of using high powers is exceedingly great; for the field of view takes in less than the diameter of the hair or wire in the finder, and the effect of the earth's diurnal motion is so great, that it requires a great deal of practice to find the object, and manage the instrument. It appears to me very probable, that the diurnal motion of the earth will be the greatest obstacle to our progress in magnifying, except we can introduce a proper mechanism to carry our telescopes in a contrary motion.

Notwithstanding opticians have proved that two eye-glasses will give a more correct image than one, I have always (from experience) persisted in refusing the assistance of a second glass, which is sure to introduce errors greater than those we would correct. Let us resign the double eye-glass to those who view objects

* For a description of this micrometer see a subsequent paper.

merely for entertainment, and must have an exorbitant field of view. To a philosopher this is an unpardonable indulgence. I have tried both the single and double eye-glasses of equal powers, and always found that the single eye-glasses had much the superiority in point of light and distinctness. With the double eye-glasses I could not see the *belts on Saturn*, which I very plainly saw with the single one. I would, however, except all those cases where a large field is absolutely necessary, and where power joined to distinctness is not the sole object of our view.

The application of the different powers of a telescope in general is of some consequence; and in answer to those who may think I have strained or over-charged mine, I must observe, that a single glance at the subsequent β Draconis, η Coronæ, and the star near μ Bootis, with a power of 460, shewed them to me as double stars; when, in two former reviews of the heavens, I had twice set them down in my journal as single stars, where I used only the power of 222 and 227, and in all probability should never have found them double, had I not looked with a higher power.

We are to remember, that it is much easier to see an object when it is pointed out to us than when it falls in our way unexpectedly, especially if of such a nature as to require some attention to be seen at all; but to say no more of other advantages of high powers, it is evident, that in the research of the parallax of the fixed stars they are absolutely necessary. If we would distinctly perceive and measure or estimate extremely small quantities, such as a tenth of a second, it appears, that when we use a power of 460, this tenth of a second will be no more in appearance than $46''$, and even with a power of 1500 will be but $2' 30''$, which is a quantity not much more than

than sufficient to judge well of objects and distinguish them from each other, such as a circle from a square, triangle, or polygon*.

It has been observed, that objects grow indistinct when the principal optic pencil at the eye becomes less than the 40th or 50th part of an inch in diameter. In the experiments that have been made upon this subject it appears to me, that the indistinctness which is ascribed to the smallness of the optical pencil may be owing to very different causes: at least it will be easy to bring contrary experiments of extremely small pencils, not at all affected by this inconvenience; for instance, it is well known, that microscopes, consisting of a single lens or globule, are remarkable for distinctness. We also know, that they have been made so small as to magnify above 10,000 times †. From this we may infer that their apertures, and consequently the diameters of the optic pencil at the eye could not exceed the 2500th part of an inch. I am therefore inclined to believe, that we must look for distinctness in the perfection of the object-speculum or object-glass of a telescope; and if we can make the first image in the focus of a speculum almost as perfect as the real object, what should hinder our magnifying but the want of light? Now, if the object has light sufficient, as the stars most undoubtedly have, I see no reason why we should limit the powers of our instruments by any theory. Is it not best to have recourse to experiments to find

* By a set of experiments, made in the year 1774, I found, that I could discover or perceive a bright object, such as white paper, against the sky-light, when it subtended an angle of 35''; but could only distinguish it to be a circle, and no other figure, when it appeared under an angle of 2' 24''.

† See Padre DELLA TORRE'S Method, &c. Scelta di Opuscoli.

how far our endeavours to render the first image perfect have been successful.

As soon as I was fully satisfied that in the investigation of parallax the method of double stars would have many advantages above any other, it became necessary to look out for proper stars. This introduced a new series of observations. I resolved to examine every star in the heavens with the utmost attention and a very high power, that I might collect such materials for this research as would enable me to fix my observations upon those that would best answer my end. The subject has already proved so extensive, and still promises so rich a harvest to those who are inclined to be diligent in the pursuit, that I cannot help inviting every lover of astronomy to join with me in observations that must inevitably lead to new discoveries. I took some pains to find out what double stars had been recorded by astronomers; but my situation permitted me not to consult extensive libraries, nor indeed was it very material: for as I intended to view the heavens myself, Nature, that great volume, appeared to me to contain the best catalogue upon this occasion. However, I remembered that the star in the head of Castor, that in the breast of the Virgin, and the first star in Aries, had been mentioned by CASSINI as double stars. I also found the Nebula in Orion was marked in HUGEN'S *Systema Saturnium* as containing seven stars, three of which (now known to be four) are very near together. With this small stock I begun, and in the course of a few years observations have collected the stars contained in my catalogue. I find, with great pleasure, that a very excellent observer, whom I have the honour to call my friend*, has also, though un-

* Phil. Transf. for the year 1781, part II. double stars discovered in 1779, at Frampton-house, Glamorganshire, by NAT. FIGOTT, Esq. F. R. S. &c.

known to me, met with three of those stars that will be found in my catalogue: and upon this occasion I also beg leave to observe, that the Astronomer Royal, when I was at Greenwich last May, with his usual politeness, shewed me, among other objects, α Herculis as a double star, which he had discovered some years ago. The rev. Mr. HORNSBY also, when I had the pleasure of seeing him at Oxford, in a conversation on the subject of the stars of the first magnitude that have a proper motion, mentioned π Bootis as a double star. It is a little hard upon young astronomers to be obliged to discover *over-again* what has already been discovered; however, the pleasure that attended the view when I first saw these stars has made some amends for not knowing they had been seen before me.

If I should mention in my list of observations a few that may be found difficult to be verified by other telescopes, I must beg the indulgence of the observers. I hope it will sufficiently appear, that I have guarded against optical delusions; and every astronomer, I make no doubt, will find, by those observations that fall within the compass of his instruments, and attention to circumstances necessary to the right management of them, that I have had all along truth and reality in view, as the sole object of my endeavours; and therefore he will be inclined to give some credit to what he does not immediately perceive, when he finds himself successful where he takes the proper precautions so necessary in delicate observations, even with the best instruments.

I have been in some doubt in what manner to communicate these observations. My first view was to have methodized them properly; but I find them so extensive that there is but little probability that one person should be able to bring them to a conclusion, for which reason I have now resolved to give them

unfinished as they are, that every person who is inclined to engage in this pursuit may become a fellow-labourer.

In settling the distances of double stars I have occasionally used two different ways. Those that are extremely near each other may be estimated by the eye, in measures of their own apparent diameters. For this purpose their distance should not much exceed two diameters of the largest, as the eye cannot so well make a good estimation when the interval between them is greater. This method has often the preference to that of the micrometer: for instance, when the diameter of a small star, perhaps not equal to half a second, is double the vacancy between the two stars. Here a micrometer ought to measure tenths of seconds at least, otherwise we could not, with any degree of confidence, rely on its measures; nay, even then, if the stars are situated in the same parallel of declination and near the equator, their quick motion across the micrometer makes it extremely difficult to measure them, and in that case an estimation by the eye is preferable to any other measure; but this requires not a little practice, precaution, and time, and yet with proper care it will be found that this method is capable of great exactness. Let two small circles be drawn either equal or unequal, at a distance not exceeding twice the diameter of the largest; let these be shewn to several persons in the same light and point of view. Then, if every one of them will separately and carefully write down his estimation of the interval between them, in the proportion of either of their diameters, it will be found upon a comparison that there will seldom be so much as a quarter of a diameter difference between all the estimations. If this agreement takes place with so many different eyes, much more may we expect it in the

estimations of the same eye when accustomed to this kind of judgement.

I have divided the double stars into several different classes. In the first I have placed all those which require indeed a very superior telescope, the utmost clearness of air, and every other favourable circumstance to be seen at all, or well enough to judge of them. They seemed to me on that account to deserve a separate place, that an observer might not condemn his instrument or his eye if he should not be successful in distinguishing them.

As these are some of the finest, most minute, and most delicate objects of vision I ever beheld, I shall be happy to hear that my observations have been verified by other persons, which I make no doubt the curious in astronomy will soon undertake. I should observe, that since it will require no common stretch of power and distinctness to see these double stars, it will therefore not be amiss to go gradually through a few preparatory steps of vision, such as the following: when η Coronæ borealis (one of the most minute double stars) is proposed to be viewed, let the telescope be some time before directed to α Geminorum, or if not in view to either of the following stars, ζ Aquarii, μ Draconis, ρ Herculis, α Piscium, or the curious double-double star ϵ Lyræ. These should be kept in view for a considerable time, that the eye may acquire the habit of seeing such objects well and distinctly. The observer may next proceed to ξ Ursæ majoris, and the beautiful treble star in Monoceros's right fore-foot; after these to i Bootis, which is a fine miniature of α Geminorum, to the star preceding α Orionis, and to ν Orionis. By this time both the eye and the telescope will be prepared for a still finer picture, which is η Coronæ borealis. It will be in vain to attempt this latter if all the former, at least

i Bootis,

i Bootis, cannot be distinctly perceived to be fairly separated because it is almost as fine a miniature of *i* Bootis as that is of α Geminorum. If the observer has been successful in all these, he may then, at the same time, try *b* Draconis, though I question whether any power less than 4 or 500 will shew it to be double; but the former I have all seen very well with 227.

To try the stars of unequal magnitudes it will be expedient to take them in some such order as the following: α Herculis, ω Aurigæ, δ Geminorum, *k* Cygni, ϵ Persei, and *b* Draconis; from these the observer may proceed to a most beautiful object, ϵ Bootis, which I have closely attended these two years as very proper for the investigation of the parallax of the fixed stars.

It appears, from what has been said, that these double stars are a most excellent way of trying a telescope; and as the foregoing remarks have suggested the method of seeing how far the power and distinctness of our instruments will reach, I shall add the way of finding how much light we have. The observer may begin with the pole-star and α Lyræ; then go to the star south of ϵ Aquilæ, the treble star near *k* Aquilæ, and last of all to the star following *o* Aquilæ. Now, if his telescope has not a great deal of good distinct light, he will not be able to see some of the small stars that accompany them.

In the second class of double stars I have put all those that are proper for estimations by the eye or very delicate measures of the micrometer. To compare the distances with the apparent diameters the power of the telescope should not be much less than 200, as they will otherwise be too close for the purpose. The instrument ought, moreover, to be as much as possible free from rays that surround a star in common telescopes, and should give the apparent diameters of a double star perfectly round and well-defined, with a deep black division between

between them, as in fig. 6. which represents α Geminorum as I have often seen it with a power of 460. It will be necessary here to take notice, that the estimations made with one telescope cannot be applied to those made with another: nor can the estimations made with different powers, though with the same telescope, be applied to each other. Whatever may be the cause of the apparent diameters of the stars, they are certainly not of equal magnitude with the same powers in different telescopes, nor of proportional magnitude with different powers in the same telescope. In my instruments I have ever found less diameter in proportion the higher I was able to go in power, and never have I found so small a proportional diameter as when I magnified 6450 times*; therefore if we would wish to compare any such observations together, with a view to see whether a change in the distance has taken place, it should be done with the very same telescope and power, even with the very same eye-glass or glasses; for others, though of equal power and goodness, would most probably give different proportional diameters of the stars.

In the third class I have placed all those double stars that are more than five but less than $15''$ asunder; and for that reason, if they should be used for observations on the parallax of the fixed stars, they ought not to be looked upon as quite free from the effects of refraction, &c. In the same manner that the stars in the first and second classes will serve to try the goodness of the most capital instruments, these will afford objects for telescopes of inferior power, such as magnify from 40 to 100 times. The observer may take them in this or the like order: ζ Ursæ majoris, γ Delphini, γ Arietis, π Bootis, γ Vir-

* See the measures of the diameter of α Lyrae. Catalogue of double stars, 5th class.

ginis, ι Cassiopeæ, μ Cygni. And if he can see all these, he may pass over into the second class, and direct his instrument to some of those that were pointed out as objects for the very best telescopes, where, I suppose, he will soon find the want of superior power.

The fourth, fifth, and sixth classes contain double stars that are from 15 to 30'', from 30'' to 1', and from 1' to 2' or more asunder. Though these will hardly be of any service for the purpose of parallax, I thought it not amiss to give an account of such as I have observed; they may, perhaps, answer another very important end, which also requires a great deal of accuracy, though not quite so much as the investigation of the parallax of the fixed stars. I will just mention it, though foreign to my present purpose. Several stars of the first magnitude have already been observed, and others suspected, to have a proper motion of their own: hence we may surmise, that our sun, with all its planets and comets, may also have a motion towards some particular part of the heavens, on account of a greater quantity of matter collected in a number of stars and their surrounding planets there situated, which may perhaps occasion a gravitation of our whole solar system towards it. If this surmise should have any foundation, it will shew itself in a series of some years; as from that motion will arise another kind of hitherto unknown parallax*, the investigation of which may account for some part of the motions already observed in some of the principal stars; and for the purpose of determining the direction and quantity of such a motion, accurate observations of the distance of stars that are near enough to be measured with a micrometer, and a very high power of

* See the note in the rev. Mr. MITCHELL's paper on the Parallax of the Fixed Stars, Phil. Transf. vol. LVII. p. 252.

telescopes may be of considerable use, as they will undoubtedly give us the relative places of those stars to a much greater degree of accuracy than they can be had by transit instruments or sectors, and thereby much sooner enable us to discover any apparent change in their situation occasioned by this new kind of systematical parallax, if I may be allowed to use that expression, for signifying the change arising from the motion of the whole solar system.

I shall now endeavour to deliver a theory of the annual parallax of double stars, with the method of computing from thence what is generally called the parallax of the fixed stars, or of single stars of the first magnitude, such as are nearest to us. It may be observed, that the principles upon which I have founded the following theory are of such a nature, that they cannot be strictly demonstrated, in consequence of which they are only proposed as postulata, which have so great a probability in their favour, that they will hardly be objected to by those who are in the least acquainted with the doctrine of chances.

GENERAL POSTULATA.

1. Let the stars be supposed, one with another, to be about the size of the sun*.

2. Let the difference of their apparent magnitudes be owing to their different distances, so that a star of the second, third,

* See Mr. MICHELL's Inquiry into the probable Parallax and Magnitude of the Fixed Stars, Phil. Transf. vol. LVII. p. 234. 236. 237. 240. and Dr. HALLEY on the Number, Order, and Light, of the Fixed Stars, Phil. Transf. vol. XXXI.

or fourth magnitude is two, three, or four times as far off as one of the first*.

In fig. 7. let OE be the whole diameter of the earth's annual orbit; and let a, b, c , be three stars situated in the ecliptic, in such a manner that they may be seen all in one line $Oabc$, when the earth is at O. Let the line $Oabc$ be perpendicular to OE, and draw PE parallel to cO . Then, if Oa, ab, bc , are equal to each other, a will be a star of the first magnitude, b of the second, and c of the third. Let us now suppose the angle OaE , or parallax of the whole orbit of the earth, to be $1''$ of a degree: then we have $PEa = OaE = 1''$: and, because very small angles, having the same subtense OE, may be taken to be in the inverse ratio of the lines Oa, Ob, Oc , &c. we shall have $ObE = \frac{1}{2}''$, $OcE = \frac{1}{3}''$, &c. †. Now, when the earth is removed

* The apparent magnitude is here taken in a stricter sense than is generally used; and by it is rather meant the order into which the stars ought to be distinguished than that into which they are commonly divided: for as the order of the magnitudes is here to denote the different relative distances, we are to examine carefully the degree of light each star is accurately found to have: and considering then that light diminishes in the inverse ratio of the squares of the distances, we ought to class the stars accordingly. An allowance ought also perhaps to be made for some loss that may happen to the light of very remote stars in its passage through immense tracts of space, most probably not quite destitute of some very subtle medium. This conjecture is suggested to us by the colour of the very small telescopic stars, for I have generally found them red, or inclining to red; which seems to indicate, that the more feeble and refrangible rays of the other colours are either stopped by the way, or at least diverted from their course by accidental deflections.

† This proves what I have before remarked on the parallax of γ Draconis; for that star, (admitting it to be a star of between the second and third magnitude, which ought to be ascertained by experiments, as mentioned in the note above) by the postulata, will have its place assigned somewhere between b and c , and therefore its parallax will be between $\frac{1}{2}$ and $\frac{1}{3}$ of the parallax of a star of the first magnitude. And if Dr. BRADLEY thought that he should have perceived a

removed to E , we shall have $PEb = EbO = \frac{1}{2}''$, and $PEa - PEb = aEb = \frac{1}{2}''$; that is, the stars a, b , will appear to be $\frac{1}{2}''$ distant. We also have $PEc = EcO = \frac{1}{3}''$, and $PEa - PEc = aEc = \frac{2}{3}''$; that is, the stars a, c , will appear to be $\frac{2}{3}''$ distant, when the earth is at E . Now, since we have $bEP = \frac{1}{2}''$, and $cEP = \frac{1}{3}''$, therefore $bEP - cEP = bEc = \frac{1}{2}'' - \frac{1}{3}'' = \frac{1}{6}''$; that is, the stars b, c , will appear to be only $\frac{1}{6}''$ removed from each other, when the earth is at E .

From what has been said, we may gather the following general expression, to denote the parallax that will become visible in the change of distance between the two stars, by the removal of the earth from one extreme of its orbit to the other. Let P express the total parallax of a fixed star of the first magnitude, M the magnitude of the largest of the two stars, m the magnitude of the smallest*, and p the partial parallax to be observed by the change in the distance of a double star; then

will $p = \frac{m-M}{Mm} P$; and p being found by observation will give

us $P = \frac{pMm}{m-M}$. An example or two will explain this sufficiently.

Suppose a star of the first magnitude should have a small star of the twelfth magnitude near it; then will the partial parallax

parallax in γ Draconis, if at most it had amounted to $2''$, it follows, that the angle OaE may nearly amount to 4 or $5''$ for any thing we can conclude to the contrary from those observations.

* As M and m are here taken to express the relative distances of the stars, in measures whereof the distance of the nearest star is taken as unity, those who think the postulata on which these estimations are built cannot be granted, may still use the following formulæ, if instead of the magnitudes M, m , they put their own estimations of the relative distances of the stars, according to any other method whatever they may think it most eligible to adopt; for the apparent magnitude of stars is here only proposed as the most probable means we have of forming any conjectures about their relative distances.

we are to expect to see be $\frac{12 \times 1}{12 - 1} P$; or $\frac{1}{11}$ ths of the total parallax of a fixed star of the first magnitude; and if we should, by observation, find the partial parallax between two such stars to amount to 1'', we shall have the total parallax $P = \frac{1 \times 1 \times 12}{12 - 1} = 1'',0909$. If the stars are of the third and twenty-fourth magnitude, the partial parallax will be $\frac{24 - 3}{3 \times 24} = \frac{21}{72} P$; and if, by observation, p is found to be a tenth of a second, the whole parallax will come out $\frac{1 \times 3 \times 24}{24 - 3} = 0'',3428$.

It will be necessary to examine some different situations. Suppose the stars, being still in the ecliptic, to appear in one line, when the earth is in any other part of its orbit between O and E; then will the parallax still be expressed by the same algebraic form, and one of the maxima will still lie at O, the other at E; but the whole effect will be divided into two parts, which will be in proportion to each other as radius - sine to radius + sine of the stars distance from the nearest conjunction or opposition.

When the stars are any where out of the ecliptic situated so as to appear in one line *Oabc* at rectangles to OE, the maximum of parallax will still be expressed by $\frac{m - M}{Mm} P$; but there will arise another additional parallax in the conjunction and opposition, which will be to that which is found 90° before or after the sun, as the sine (S) of the latitude of the stars seen at O is to radius (R); and the effect of this parallax will be divided into two parts; half of it lying on one side of the large star, the other half on the other side of it. This latter parallax, moreover, will be compounded with the former, so that

the distance of the stars in the conjunction and opposition will then be represented by the diagonal of a parallelogram, whereof the two semi-parallaxes are the sides; a general expression for which will be $\sqrt{\frac{m-M}{2Mm} P^2 \times \frac{SS}{RR} + 1}$: for the stars will apparently describe two ellipses in the heavens, whose transverse axes will be to each other in the ratio of M to m (fig. 8.), and Aa , Bb , Cc , Dd , will be cotemporary situations. Now, if bQ be drawn parallel to AC , and the parallelogram $bqBQ$ completed, we shall have $bQ = \frac{1}{2}CA - \frac{1}{2}ca = \frac{1}{2}Cc = \frac{1}{2}p$, or semi-parallax 90° before or after the sun, and Bb may be resolved into, or is compounded of, bQ and bq ; but $bq = \frac{1}{2}BD - \frac{1}{2}bd =$ the semi-parallax in the conjunction or opposition. We also have $R : S :: bQ : bq = \frac{pS}{2R}$; therefore the distance Bb (or Dd) =

$\sqrt{\left[\frac{p}{2}\right]^2 + \left[\frac{pS}{2R}\right]^2}$; and by substituting the value of p into this ex-

pression we obtain $\sqrt{\frac{m-M}{2Mm} P^2 \times \frac{SS}{RR} + 1}$, as above. When the stars are in the pole of the ecliptic, bq will become equal to bQ , and Bb will be $.7071 P \frac{m-M}{Mm}$.

Hitherto we have supposed the stars to be all in one line $Oabc$; let them now be at some distance, suppose $5''$ from each other, and let them first be both in the ecliptic. This case is resolvible into the first; for imagine the star a , fig. 9. to stand at x , and in that situation the stars x , b , c , will be in one line, and their parallax expressed by $\frac{m-M}{Mm} P$. But the angle aEx may be taken to be equal to aOx ; and as the foregoing form gives us the angles xEb , xEc , we are to add aEx , or $5''$ to xEb , and we shall have aEb . In general, let the distance

tance of the stars be d , and let the observed distance at E be D ; then will $D = d + p$, and therefore the whole parallax of the annual orbit will be expressed by $\frac{DMm - dMm}{m - M} = P$.

Suppose the two stars now to differ only in latitude, one being in the ecliptic, the other, for instance, $5''$ north, when seen at O . This case may also be resolved by the former; for imagine the stars b , c , fig. 7. to be elevated at rectangles above the plane of the figure, so that aOb , or aOc , may make an angle of $5''$ at O : then, instead of the lines $Oabc$, Ea , Eb , Ec , EP , imagine them all to be planes at rectangles to the figure; and it will appear, that the parallax of the stars in longitude must be the same as if the small star had been without latitude. And since the stars b , c , by the motion of the earth from O to E , will not change their latitude, we shall have the following construction for finding the distance of the stars ab , ac , at E , and from thence the parallax P . Let the triangle $ab\beta$, fig. 10. represent the situation of the stars; ab is the subtense of $5''$, that being the angle under which they are supposed to be seen at O . The quantity $b\beta$ by the former theorem is found $\frac{m - M}{Mm} P$, which is the partial parallax that would have been seen by the earth's moving from O to E , had both stars been in the ecliptic; but on account of the difference in latitude it will now be represented by $a\beta$, the hypotenuse of the triangle $ab\beta$: therefore, in general, putting $ab = d$, and $a\beta = D$, we have $\frac{\sqrt{DD - dd} \times Mm}{m - M} = P$. Hence D being taken by observation and d , M , and m , given, we obtain the total parallax.

If the situation of the stars differs in longitude as well as latitude, we may resolve this case by the following method.

Let

Let the triangle $ab\beta$, fig. 11. represent the situation of the stars, $ab = d$ being their distance seen at O, $a\beta = D$ their distance seen at E. That the change $b\beta$ which is produced by the earth's motion will be truly expressed by $\frac{m-M}{Mm} P$, may be proved as before, by supposing the star a to have been placed at α . Now let the angle of position $ba\alpha$ be taken by a micrometer*, or by any other method that may be thought sufficiently exact; then, by solving the triangle $ab\alpha$, we shall have the longitudinal and latitudinal differences $a\alpha$ and $b\alpha$ of the two stars. Put $a\alpha = x$, $b\alpha = y$, and it will be $x + b\beta = aq$, whence $D = \sqrt{x + \frac{m-M}{Mm} P} + yy$; and $\frac{\sqrt{D^2 - y^2} \times M^2 m^2 - xMm}{m-M} = P$.

If neither of the stars should be in the ecliptic, nor have the same longitude or latitude, the last theorem will still serve to calculate the total parallax whose maximum will lie in E. There will, moreover, arise another parallax, whose maximum will be in the conjunction and opposition, which will be divided, and lie on different sides of the large star; but as we know the whole parallax to be exceedingly small, it will not be necessary to investigate every particular case of this kind; for, by reason of the division of the parallax, which renders observations taken at any other time, except where it is greatest, very unfavourable, the forms would be of little use.

To finish this theory, I shall only add a general observation on the time and place where the maxima of parallax will happen.

* The position of a line passing through the two stars, with the parallel of declination of the largest of them, may be had by the micrometer I invented for this purpose in the year 1779, of which a description has been given in a former paper; whence, by spherical trigonometry, we easily deduce their position $ba\alpha$ fig. 11. with regard to the ecliptic.

When

Fig. 1.

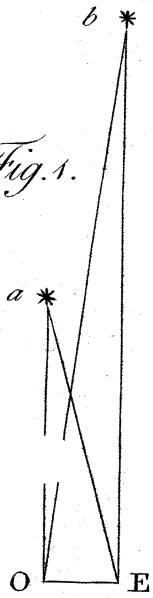


Fig. 2.



Fig. 3.

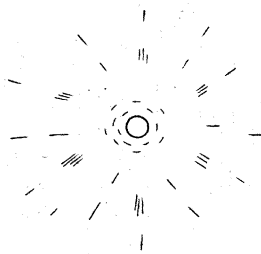


Fig. 5.

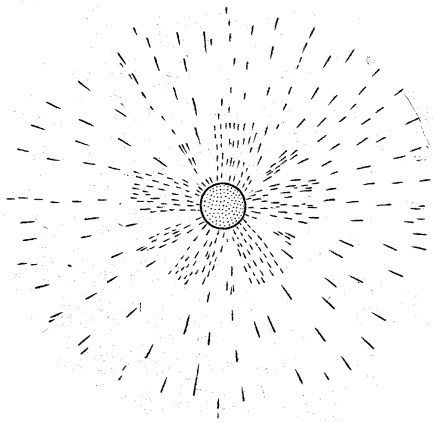


Fig. 6.



Fig.

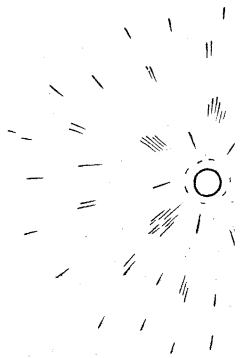


Fig. 4.

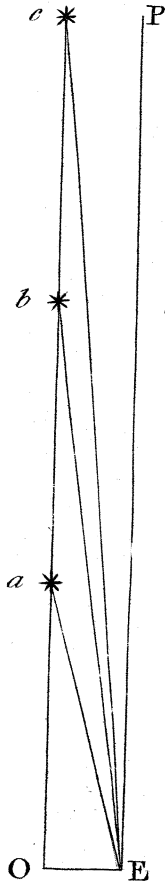
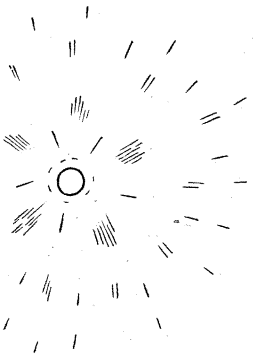


Fig. 7.

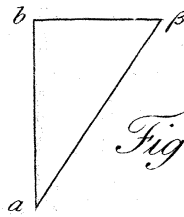


Fig. 10.

Fig. 8.

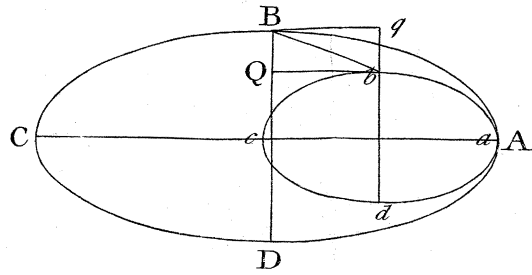


Fig. 11.

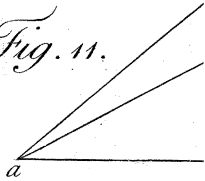
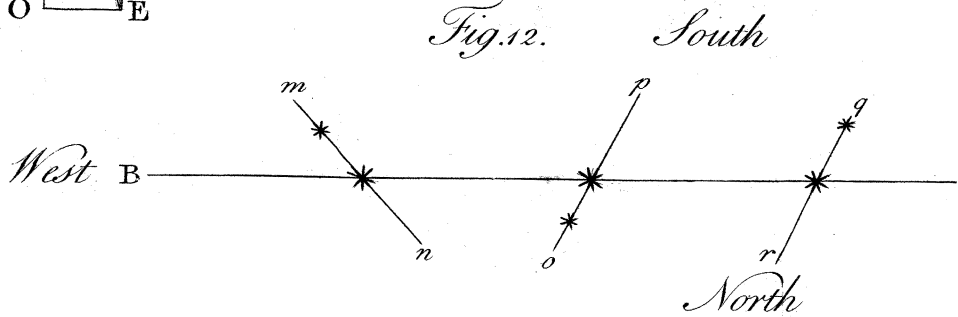


Fig. 12.



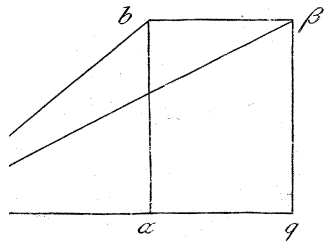
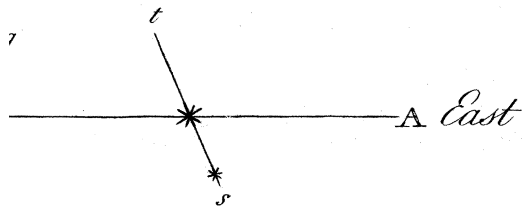
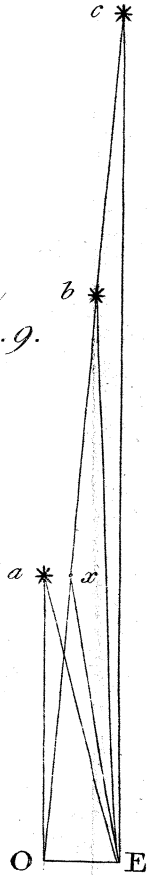


Fig. 9.



When two unequal stars are both in the ecliptic, or, not being in the ecliptic, have equal latitudes, north or south, and the largest star has most longitude, the maximum of the apparent distance will be when the sun's longitude is 90° more than the stars, or when observed in the morning; and the minimum when the longitude of the sun is 90° less than that of the star, or when observed in the evening.

When the small star has most longitude, the maximum and minimum, as well as the time of observation, will be the reverse of the former.

When the stars differ in latitudes, this makes no alteration in the place of the maximum or minimum, nor in the time of observation; that is to say, it is immaterial whether the largest star has the least or the most latitude of the two stars.



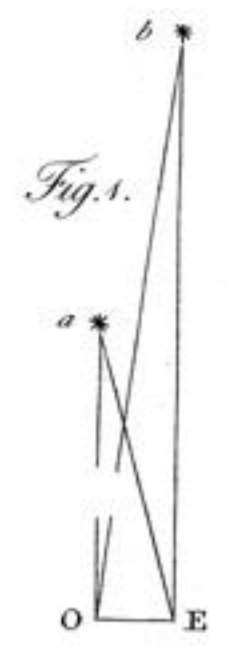


Fig. 1.



Fig. 2.



Fig. 3.

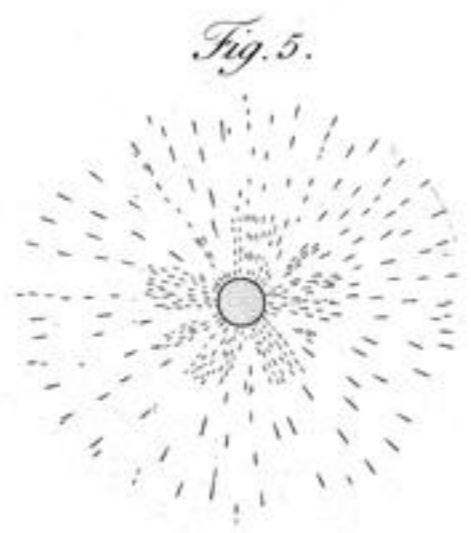


Fig. 5.

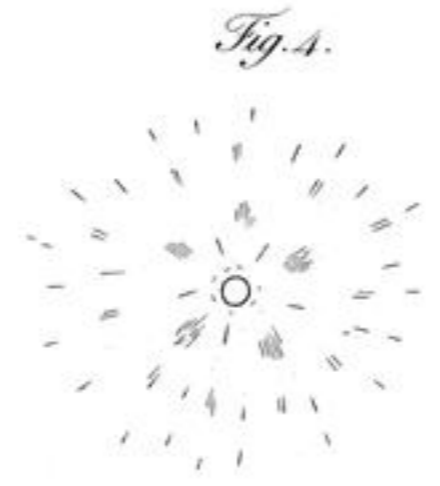


Fig. 4.



Fig. 6.



Fig. 7.

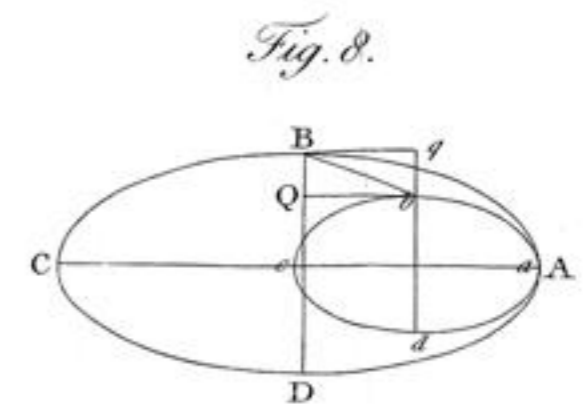


Fig. 8.

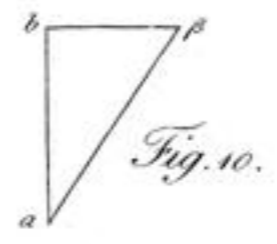


Fig. 10.

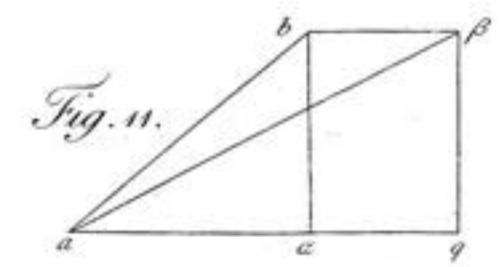


Fig. 11.



Fig. 9.

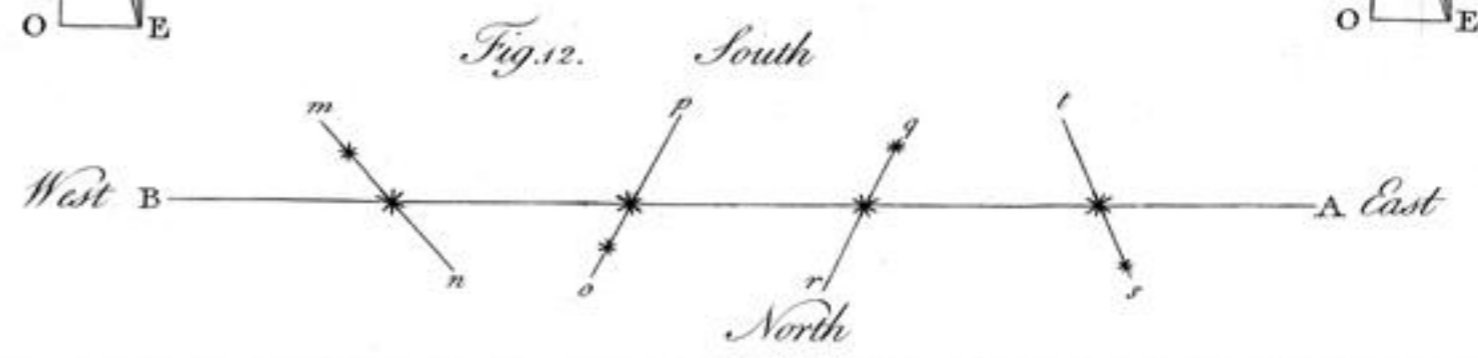


Fig. 12.